BUFFALO ASTRONOMICAL ASSOCIATION INC.
BUFFALO MUSEUM OF SCIENCE
HUMBOLDT PARKWAY
BUFFALO NEW YORK 14211

the Spectrum

Editor:
Ernst E. Both

JUNE 1971

ANNUAL MEETING, June 11, 1971: Our annual meeting will take place on June 11, 1971, at 8:00 PM, EDT, at the Buffalo Museum of Science. The program will consist of the following: Annual Report by the President, Mr. Zygmunt; election of three members to the Board of Directors; ratification of the election of three members to the College of Fellows; and an illustrated lecture-demonstration on "Image Intensifiers in Astronomy" by our own Bill Chambers.

The following members are candidates for election to the Board: Robert Burdick, William Chambers, Irving Goetz, Mrs. Sylvia Mosure, Dr. Fred Price, and Darl Washburn.

Ratification of the election of the following members to the College of Fellows (see The Spectrum, October 1970): Ronald Clippinger, Walter Whyman, and Richard Zygmunt. According to our By-Laws, "to be elected a Fellow, a member must be sponsored by two Fellows, and elected by a majority of the entire College of Fellows. Election must be approved by the Board of Directors and ratified by a majority of Ordinary Members present at the annual meeting." The above-named gentlemen were elected by a majority of Fellows and their election has been approved by the Board of Directors.

This is an important meeting, so come and vote! We are happy to welcome as our guest speaker BILL CHAMBERS!

* * *

SUMMER STAR NIGHTS: As is our custom, we take "to the fields" during the summer for week-end observation meetings. These star nights are usually held on Fridays (starting at dusk) or, if cloudy, on the following Saturday. This is a preliminary list (a complete list with maps will be mailed around mid-June): July 2/3, Les Stoklosa's summer home, Colden, N.Y.; July 9/10, Newstead Observatory; July 16/17 Camp Sprucelands; July 23/24 Irving Goetz's home, Hamburg, N.Y.; July 30/31 Newstead Observatory; August 6/7 Kellogg Observatory (Mars); August 13/14 Larry Hazel's home, Niagara Falls, N.Y.; August 20/21 Camp Sprucelands; August 27/28 The Geiger's, N. Boston, N.Y.

*

* NOTE ON THE FORMULA FOR A MIRROR'S RADIUS OF CURVATURE * By Frederick R. West

In the April 1971 issue of The Spectrum, Robert Burdick ("Measuring a Mirror's Radius of Curvature" pag. 1) quoted a formula for the radius of curvature R of a mirror which should read, when correctly typed,

$$R = r + 7P^2 \qquad (1)$$

where r is the radius of a ball rolling on the mirror's curved surface and P is the period of time for the ball to roll across the mirror's surface and back to its initial position. In equation (1) the units of R and r are inches, and P is in seconds of time. While reading Burdick's interesting article, I became curious about the physical basis for equation (1) and after some thought I was able to derive this equation from the laws of mechanics.

The derivation will start with Figure 1. Here we see part of the cross-section EC of a curved mirror whose radius of curvature is R with its center of curvature at O. A homogeneous solid ball whose center O' is shown in Figure 1 is in contact with the mirror surface at C and rolls along the mirror's curved surface while acted on by the forces shown in Figure 1. We show two co-ordinate axes: the y axis, which coincides with the optical axis of the mirror and is positive in the direction of O, its center of curvature, and the x axis, which passes through E, the center of the mirror surface, is positive to the right of E, and is perpendicular to the y axis. E is the origin of the co-ordinate system. Consider the position, motion, and acceleration a_x of the center of mass O' of the ball of mass m and radius r. Writing Newton's second law for the ball along the x co-ordinate,

$$ma_x = F_x \qquad (2)$$

we see that the forces on the ball along the x direction F_x are:

$$F_x = (-mg \sin\theta + F) \cos\theta \qquad (3)$$

where g is the acceleration of gravity and F is the force of friction acting at point C between the ball and the mirror surface. In this derivation we assume that the mirror is horizontal, and therefore that the y axis points to the zenith. The angle θ is the angle that the mirror surface radius OC makes with the y axis.

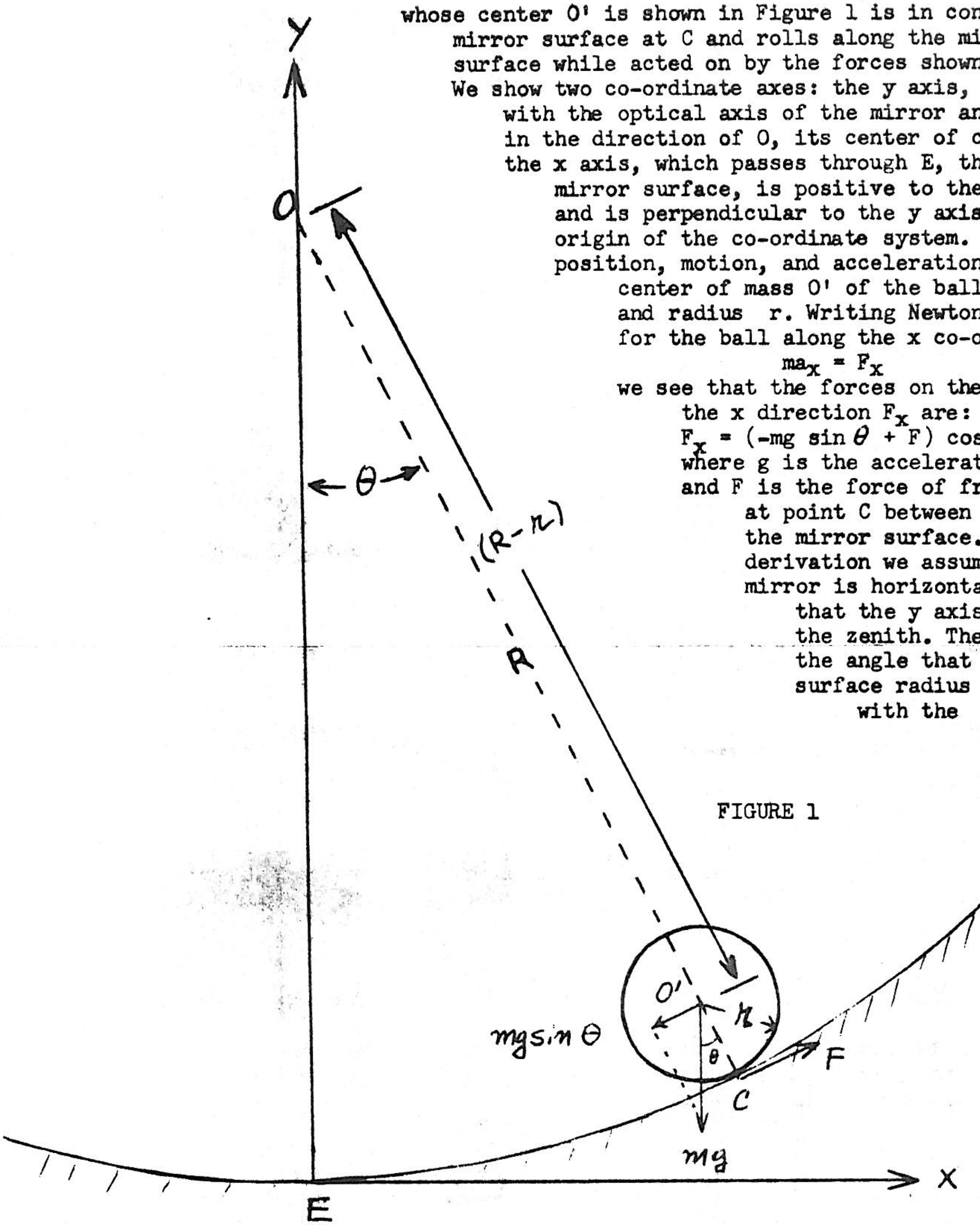


FIGURE 1

If the ball is rolling (with no sliding) along the mirror surface, we can write the following equations for the torque τ about O' on the ball:

$$\tau = I\alpha \qquad (4a)$$
$$\tau = Fr \qquad (4b)$$

where α is the angular acceleration of the ball and I is its moment of inertia. Furthermore, we see from geometry in Figure 1

$$\alpha r \cos\theta = -a_x \qquad (5).$$

For a homogeneous solid sphere, $I = \frac{2}{5} mr^2$ (6).

Equating equations (4a) and (4b), then substituting for I and α from equations (6) and (5) respectively, we find

$$\frac{2}{5} mr^2 \left(- \frac{a_x}{r \cos\theta} \right) = Fr$$

$$- \frac{2}{5} m \frac{a_x}{\cos\theta} = F$$

Here and in equation (5) the minus sign is used because an angular acceleration in Figure 1 is associated with a deceleration (negative acceleration) along the x axis.

Substituting for F in equations (3) and (2) we get:

$$ma_x = -mg \sin\theta \cos\theta - \frac{2}{5} ma_x$$

$$\frac{7}{5} ma_x = -mg \sin\theta \cos\theta \qquad (7).$$

If θ is smaller than 5°, as is the case for mirror focal-ratios of 3 or greater, we can approximate $\cos\theta$ by

$$\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \ldots\ldots \qquad (8)$$

and with good accuracy (since all but the first term are less than 0.01) we can write $\cos\theta \simeq 1$. Equation (7) then becomes

$$ma_x = -\frac{5}{7} mg \sin\theta$$

and since from the geometry of Figure 1 for point O'

$$\sin\theta = \frac{x}{R - r} \qquad (9)$$

$$ma_x = -\frac{5}{7} mg \frac{x}{R - r}$$

or dividing out m,

$$a_x = -\frac{5}{7} \frac{g}{R - r} x \qquad (10)$$

so that within the accuracy of the approximations made, the acceleration a_x is directly proportional to and opposite in sign to the displacement x of the ball. There is a well-known class of systems in mechanics for which a similar relation holds between a_x and x, and the motion that such systems describe is known as simple harmonic motion. The simple pendulum and a mass oscillating at the end of a spring are perhaps the two most common and well-known examples of systems which perform simple harmonic motion, whose displacement x in one co-ordinate with time varies (ignoring friction) as a sine or cosine function of a system constant, times the time. For example, from equation (10) we could, given suitable boundary conditions, derive and write the following equation for x:

$x = A \cos \sqrt{\frac{5}{7}\frac{g}{R - r}}\; t$ .....(11) where A is the ball's amplitude of roll across the mirror, and t is the time. Here t=0 is taken to be an instant of maximum x displacement of the ball. The period P for the ball to oscillate across the mirror and back to its initial position on the mirror is given by the formula

$P = 2\pi\sqrt{\frac{7}{5}\frac{R - r}{g}}$ .....(12) which is solved for (R-r):

$$R - r = \frac{5}{7}\frac{g}{4\pi^2} P^2 \qquad (12a)$$

If one sets g= 386.4 inches/sec^2 to get lengths in inches and time in seconds, then equation (12a) becomes

$$(R - r) = 7P^2 \qquad (12b)$$

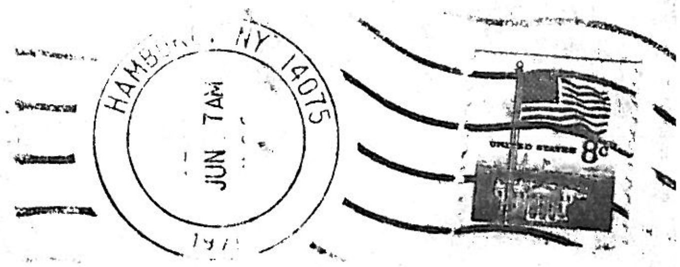
which is the same as equation (1).

* * *

SUMMER SEMINAR, Syracuse Astronomical Society Science Center Observatory, Saturday July 17, 1971.Syracuse Astronomical Society has just opened its new Science Center Observatory, complete with a new 16-inch, f/6 reflector. The observatory is located off Strong Rd., south of the Village of Vesper, N.Y., in Tully Township, about 20 min. drive from Syracuse (we will have a map in the July-Agust issue of The Spectrum). The Society hopes to make this an annual affair, a sort of New York equivalent of Stellafane. Those who are interested in attending should write to: Syracuse Astron. Society, 1115 E Colvin St., Syracuse, N.Y. 13210, attention: Joseph Italiano, Summer Seminar Chairman. Papers are invited, as well as telescopes and/or equipment. ****

*

The Buffalo Astronomical Association, Inc.
c/o Buffalo Museum of Science
Buffalo, N.Y. 14211



FIRST CLASS

Mr. Walter Whyman
193 Oak St.
Batavia, N. Y.
14020